

# The Jackiw-Pi model: classical theory

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(Dated: September 26, 2012)

The massive even-parity non-Abelian gauge model in three space-time dimensions proposed by Jackiw and Pi is studied at the tree-level. The propagators are computed and the spectrum consistency is analyzed, besides, the symmetries of the model are collected and established through BRS invariance and Slavnov-Taylor identity. In the Landau gauge, thanks to the antighost equations and the Slavnov-Taylor identity, two rigid symmetries are identified by means of Ward identities. It is presented here a promising path for perturbatively quantization of the Jackiw-Pi model and a hint concerning its possible quantum scale invariance is also pointed out.

In honor of the 70th birthday of Prof. Olivier Piguet

PACS numbers: 11.10.Gh, 11.15.-q, 11.15.Bt, 11.15.Ex

## I. INTRODUCTION

The study of gauge field theories in three space-time dimensions has raised a great deal of interest since the early works of Deser, Jackiw and Templeton [1]. Over the last decades, this issue has also been motivated and well-supported in view of the possibilities they open up for the setting of a gauge field theoretical foundation in the description of condensed matter phenomena, such as high- $T_c$  superconductivity and quantum Hall effect. Meantime, one of the central problems in the framework of gauge field theories is the issue of gauge field mass. Gauge symmetry is not, in principle, conflicting with the presence of a massive gauge boson. In two space-time dimensions, the well-known Schwinger model puts in evidence the presence of a massive photon without the breaking of gauge symmetry [2]. Another evidence for the compatibility between gauge symmetry and massive vector fields has been arisen in the study of three-dimensional gauge theories, when a topological mass term referred to as the Chern-Simons one, once added to the Yang-Mills term, shifts the photon mass to a non-vanishing value without breaking gauge invariance, however parity symmetry is lost [1]. Nevertheless, Jackiw and Pi overcame the challenge to implement both gauge and parity invariance in three space-time dimensions by breaking the Yang-Mills paradigm - non-Abelian generalizations of Abelian models. They proposed a three-dimensional non-Yang-Mills gauge model for a pair of vector fields with opposite parity transformations, which generates a mass-gap through a mixed Chern-Simons-like term preserving parity [3]. The Jackiw-Pi model has also been studied in the Hamiltonian framework [4], where physical states consistency was demonstrated [4]. Recently, by using the BRS approach, new symmetries and gauge-fixing were established [5], and in [6] the Yang-Mills symmetry sector was analyzed through the Bonora-Tonin superfield formalism [7]. The Jackiw-Pi model remains unquantized up to now, however, it is presented here the key ingredients for its further perturbatively quantization [8] through the algebraic method of renormalization [9]. In this work, the non-Abelian gauge model proposed by Jackiw and Pi, which generates an even-parity mass term in three space-time dimensions, is revisited. The model and its gauge symmetries, the BRS symmetry, the gauge-fixing and the anti-fields action are presented in Section II. The BRS approach has allowed to bypass the difficulties addressed in the literature with respect to the gauge-fixing. In Section III, the tree-level propagators are computed, the spectrum consistency (causality and unitarity) is analyzed and the ultraviolet and infrared dimensions of all the fields are established. In the Section IV, the Slavnov-Taylor identity, ghost and anti-ghost equations, and the operatorial algebra are presented. Furthermore, it is shown that in the Landau gauge, thanks to the antighost equations and the Slavnov-Taylor identity, two rigid symmetries of the Jackiw-Pi model are identified by means of Ward identities.

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## II. THE MODEL AND ITS SYMETRIES

### A. The model

The classical action of the Jackiw-Pi model [3] is given by:

$$\Sigma_{\text{inv}} = \text{Tr} \int d^3x \left\{ \frac{1}{2} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} (G^{\mu\nu} + g[F^{\mu\nu}, \rho]) (G_{\mu\nu} + g[F_{\mu\nu}, \rho]) - m \epsilon^{\mu\nu\rho} F_{\mu\nu} \phi_\rho \right\} , \quad (1)$$

such that,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu] , \quad G_{\mu\nu} = D_\mu \phi_\nu - D_\nu \phi_\mu \quad \text{and} \quad D_\mu \bullet = \partial_\mu \bullet + g[A_\mu, \bullet] , \quad (2)$$

where  $A_\mu$  and  $\phi_\mu$  are vector fields,  $\rho$  is a scalar,  $g$  is a coupling constant and  $m$  a mass parameter, also,  $\bullet$  means any field. The Lie group is a simple compact, so that every field,  $X = X_a \tau_a$ , is Lie algebra valued, with the matrices  $\tau$  being the generators of the group in the adjoint representation and obey

$$[\tau_a, \tau_b] = f_{abc} \tau_c \quad \text{and} \quad \text{Tr}(\tau_a \tau_b) = -\frac{1}{2} \delta_{ab} \quad (a, b, c = 1, 2, \dots, N^2 - 1) . \quad (3)$$

### B. Gauge symmetries

The action (1) is invariant under two sets of gauge transformations,  $\delta_\theta$  and  $\delta_\chi$ :

$$\delta_\theta A_\mu = D_\mu \theta , \quad \delta_\theta \phi_\mu = g[\phi_\mu, \theta] \quad \text{and} \quad \delta_\theta \rho = g[\rho, \theta] ; \quad (4)$$

$$\delta_\chi A_\mu = 0 , \quad \delta_\chi \phi_\mu = D_\mu \chi \quad \text{and} \quad \delta_\chi \rho = -\chi , \quad (5)$$

where  $\theta$  and  $\chi$  are Lie algebra valued infinitesimal local parameters.

### C. BRS symmetry

The corresponding BRS transformations of the fields  $A_\mu$ ,  $\phi_\mu$  and  $\rho$ , stemming from the symmetries (4) and (5), are given by<sup>1</sup>:

$$\begin{aligned} sA_\mu &= D_\mu c , \quad s\phi_\mu = D_\mu \xi + g[\phi_\mu, c] , \quad s\rho = -\xi + g[\rho, c] , \\ sc &= -gc^2 \quad \text{and} \quad s\xi = -g[\xi, c] , \end{aligned} \quad (6)$$

where  $c$  and  $\xi$  are the Faddeev-Popov ghosts, with Faddeev-Popov charge (ghost number) one. The ghost number  $(\Phi\Pi)$  of all fields and antifields are collected in Table II

### D. The gauge-fixing and the antifields action

The gauge-fixing adopted here belongs to the class of the linear covariant gauges discussed by 't Hooft [10]. In order to implement the gauge-fixing following the BRS procedure [11], we introduce two sorts of ghosts ( $c$  and  $\xi$ ), antighosts ( $\bar{c}$  and  $\bar{\xi}$ ) and the Lautrup-Nakanishi fields [12] ( $b$  and  $\pi$ ), playing the role of Lagrange multiplier fields for the gauge condition, such that

$$s\bar{c} = b , \quad sb = 0 ; \quad (7)$$

$$s\bar{\xi} = \pi , \quad s\pi = 0 ; \quad (8)$$

where the multiplier fields,  $b$  and  $\pi$ , and the Faddeev-Popov antighosts,  $\bar{c}$  and  $\bar{\xi}$ , with ghost number minus one, belong to the BRS-doublets (7) and (8).

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<sup>1</sup> The commutators among the fields are assumed to be graded, namely,  $[\varphi_1^{g_1}, \varphi_2^{g_2}] \equiv \varphi_1^{g_1} \varphi_2^{g_2} - (-1)^{g_1 \cdot g_2} \varphi_2^{g_2} \varphi_1^{g_1}$ , where the upper indices,  $g_1$  and  $g_2$ , are the Faddeev-Popov charges  $(\Phi\Pi)$  carried by the fields  $\varphi_1^{g_1}$  and  $\varphi_2^{g_2}$ , respectively.

Now, by adopting the gauge conditions

$$\frac{\delta \Sigma_{\text{gf}}}{\delta b} = \partial^\mu A_\mu + \alpha b , \quad (9)$$

$$\frac{\delta \Sigma_{\text{gf}}}{\delta \pi} = \partial^\mu \phi_\mu + \beta \pi , \quad (10)$$

it follows that the BRS-trivial gauge-fixing action compatible with then reads

$$\begin{aligned} \Sigma_{\text{gf}} &= s \text{Tr} \int d^3x \left\{ \bar{c} \partial^\mu A_\mu + \bar{\xi} \partial^\mu \phi_\mu + \frac{\alpha}{2} \bar{c} b + \frac{\beta}{2} \bar{\xi} \pi \right\} \\ &= \text{Tr} \int d^3x \left\{ b \partial^\mu A_\mu - \bar{c} \partial^\mu D_\mu c + \pi \partial^\mu \phi_\mu - \bar{\xi} \partial^\mu (D_\mu \xi + g[\phi_\mu, c]) + \frac{\alpha}{2} b^2 + \frac{\beta}{2} \pi^2 \right\} . \end{aligned} \quad (11)$$

Let us now introduce the action in which the nonlinear BRS transformations are coupled to the antifields (BRS invariant external fields), so as to control, at the quantum level, the renormalization of those transformations:

$$\Sigma_{\text{ext}} = \text{Tr} \int d^3x \left\{ A_\mu^* s A^\mu + \phi_\mu^* s \phi^\mu + \rho^* s \rho + c^* s c + \xi^* s \xi \right\} , \quad (12)$$

where, as mentioned above, the antifields are BRS invariant, namely,

$$s A_\mu^* = s \phi_\mu^* = s \rho^* = s c^* = s \xi^* = 0 . \quad (13)$$

The total action at the tree level for the Jackiw-Pi model,  $\Gamma^{(0)}$ , is therefore given by:

$$\Gamma^{(0)} = \Sigma_{\text{inv}} + \Sigma_{\text{gf}} + \Sigma_{\text{ext}} , \quad (14)$$

which is invariant under the BRS transformations given by the equations (6), (7), (8) and (13). The action (14) preserves the ghost number. The values of the ghost number, the ultraviolet (UV) and the infrared (IR) dimensions (respected to the Landau gauge) are displayed in Table II - all subtleties concerning the determination of the UV and the IR dimensions of the fields, in the Landau gauge, is presented in the next section. The statistics is defined as follows: the fields of integer spin and odd ghost number as well as the fields of half integer spin and even ghost number are anticommuting; the other fields commute with the formers and among themselves.

An interesting feature of the Jackiw-Pi action  $\Gamma^{(0)}$  (14) is that it is not BRS local invariant thanks to the parity-even mass term:

$$\Sigma_m = \text{Tr} \int d^3x \left\{ -m \epsilon^{\mu\nu\rho} F_{\mu\nu} \phi_\rho \right\} , \quad (15)$$

since

$$s F_{\mu\nu} = g[F_{\mu\nu}, c] , \quad (16)$$

then

$$s \Sigma_m = -m s \text{Tr} \int d^3x \left\{ \epsilon^{\mu\nu\rho} F_{\mu\nu} \phi_\rho \right\} = -m \text{Tr} \int d^3x \left\{ \epsilon^{\rho\mu\nu} \partial_\rho (F_{\mu\nu} \xi) \right\} , \quad (17)$$

which is invariant only up to a total derivative, possibly indicating that at the quantum level the  $\beta$ -function associated to the mass parameter  $m$  vanishes [13, 14].

### III. SPECTRAL ANALYSIS

In quantum field theory, unitarity and causality are essential physical requirements. Unitarity (of the  $S$ -matrix) reflects the fundamental principle of probability conservation – meaning the absence of negative-norm 1-particle states in the spectrum. Even though we have to introduce in certain instances the artificial device of an indefinite metric in Hilbert space, the physical quantities always refer to positive-norm states, preserved through the time evolution. Causality principle establishes a time correlation among the cause and its subsequent effect, requiring that the change in the interaction law in any space-time region can influence the evolution of the system only at subsequent times.

### A. The propagators

The propagators are the key ingredient to the analysis of the spectral consistency and the unitarity at the tree-level of the model, as well as in the determination of the ultraviolet ( $d$ ) and infrared ( $r$ ) dimensions of the fields.

By switching off the coupling constant  $g$  we get the free part of the action,  $\Sigma_{\text{inv}} + \Sigma_{\text{gf}}$ , as follows:

$$\begin{aligned} \Sigma_{\text{free}} = & \text{Tr} \int d^3x \left\{ \frac{1}{2} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} G^{\mu\nu} G_{\mu\nu} - m \epsilon^{\mu\nu\rho} F_{\mu\nu} \phi_\rho + b \partial^\mu A_\mu + \frac{\alpha}{2} b^2 + \pi \partial^\mu \phi_\mu + \frac{\beta}{2} \pi^2 + \right. \\ & \left. - \bar{c} \partial^\mu \partial_\mu c - \bar{\xi} \partial^\mu \partial_\mu \xi \right\}, \end{aligned} \quad (18)$$

where, by means of the operators:

$$\Theta^{\mu\nu} = \eta^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{\square}, \quad \Omega^{\mu\nu} = \frac{\partial^\mu \partial^\nu}{\square} \quad \text{and} \quad \Sigma^{\mu\nu} = \epsilon^{\mu\rho\nu} \partial_\rho, \quad (19)$$

that fulfil the algebra displayed in Table I, the free action  $\Sigma_{\text{free}}$ (18) can be written as:

$$\begin{aligned} \Sigma_{\text{free}} = & \text{Tr} \int d^3x \left\{ -A_\mu \square \Theta^{\mu\nu} A_\nu - \phi_\mu \square \Theta^{\mu\nu} \phi_\nu - 2m A_\mu \Sigma^{\mu\nu} \phi_\nu + b \partial^\mu A_\mu + \frac{\alpha}{2} b^2 + \pi \partial^\mu \phi_\mu + \frac{\beta}{2} \pi^2 + \right. \\ & \left. - \bar{c} \square c - \bar{\xi} \square \xi \right\}, \\ = & \int d^3x \left\{ \frac{1}{2} A_\mu^a \square \Theta^{\mu\nu} A_\nu^a + \frac{1}{2} \phi_\mu^a \square \Theta^{\mu\nu} \phi_\nu^a + m A_\mu^a \Sigma^{\mu\nu} \phi_\nu^a - \frac{1}{2} b^a \partial^\mu A_\mu^a - \frac{\alpha}{4} b^a b^a - \frac{1}{2} \pi^a \partial^\mu \phi_\mu^a - \frac{\beta}{4} \pi^a \pi^a + \right. \\ & \left. + \frac{1}{2} \bar{c}^a \square c^a + \frac{1}{2} \bar{\xi}^a \square \xi^a \right\}. \end{aligned} \quad (20)$$

The generating functional for the connected Green functions ( $Z^c[J]$ ) is defined by means of the vertex functional ( $\Gamma^{(0)}$ ) through the Legendre transformation [15]:

$$Z^c[J_i] = \Gamma^{(0)}[\Phi_i] + \text{Tr} \int d^3x \left( A_\mu J_A^\mu + \phi_\mu J_\phi^\mu + b J_b + \pi J_\pi + \bar{J}_c c + J_{\bar{c}} \bar{c} + \bar{J}_\xi \xi + J_{\bar{\xi}} \bar{\xi} \right), \quad (21)$$

where  $\Phi_i = (A_\mu, \phi_\mu, b, \pi, c, \bar{c}, \xi, \bar{\xi})$  and  $J_i = (J_A^\mu, J_\phi^\mu, J_b, J_\pi, \bar{J}_c, J_{\bar{c}}, \bar{J}_\xi, J_{\bar{\xi}})$ , such that

$$\begin{aligned} \frac{\delta Z^c}{\delta J_A^\mu(x)} &= A_\mu(x), \quad \frac{\delta \Gamma^{(0)}}{\delta A_\mu(x)} = -J_A^\mu(x), \quad \frac{\delta Z^c}{\delta J_\phi^\mu(x)} = \phi_\mu(x), \quad \frac{\delta \Gamma^{(0)}}{\delta \phi_\mu(x)} = -J_\phi^\mu(x), \\ \frac{\delta Z^c}{\delta J_b(x)} &= b(x), \quad \frac{\delta \Gamma^{(0)}}{\delta b(x)} = -J_b(x), \quad \frac{\delta Z^c}{\delta J_\pi(x)} = \pi(x), \quad \frac{\delta \Gamma^{(0)}}{\delta \pi(x)} = -J_\pi(x), \\ \frac{\delta Z^c}{\delta \bar{J}_c(x)} &= c(x), \quad \frac{\delta \Gamma^{(0)}}{\delta \bar{c}(x)} = \bar{J}_c(x), \quad \frac{\delta Z^c}{\delta J_{\bar{c}}(x)} = \bar{c}(x), \quad \frac{\delta \Gamma^{(0)}}{\delta \bar{c}(x)} = J_{\bar{c}}(x), \\ \frac{\delta Z^c}{\delta \bar{J}_\xi(x)} &= \xi(x), \quad \frac{\delta \Gamma^{(0)}}{\delta \bar{\xi}(x)} = \bar{J}_\xi(x), \quad \frac{\delta Z^c}{\delta J_{\bar{\xi}}(x)} = \bar{\xi}(x), \quad \frac{\delta \Gamma^{(0)}}{\delta \bar{\xi}(x)} = J_{\bar{\xi}}(x). \end{aligned} \quad (22)$$

The tree-level propagators for all the fields:

$$\langle T \Phi_i(x) \Phi_j(y) \rangle = -i \frac{\delta^2 Z^c}{\delta J_i(x) \delta J_j(y)}, \quad (23)$$

are then computed, through the use of Eq.(22), as follows:

$$\begin{aligned} \langle T A_\mu^a(x) A_\nu^b(y) \rangle &= -i \frac{\delta A_\mu^a(x)}{\delta J_\nu^{b\nu}(y)}, \quad \langle T \phi_\mu^a(x) \phi_\nu^b(y) \rangle = -i \frac{\delta \phi_\mu^a(x)}{\delta J_\nu^{b\nu}(y)}, \quad \langle T A_\mu^a(x) \phi_\nu^b(y) \rangle = -i \frac{\delta A_\mu^a(x)}{\delta J_\nu^{b\nu}(y)}, \\ \langle T A_\mu^a(x) b^b(y) \rangle &= -i \frac{\delta A_\mu^a(x)}{\delta J_b^b(y)}, \quad \langle T \phi_\mu^a(x) \pi^b(y) \rangle = -i \frac{\delta \phi_\mu^a(x)}{\delta J_\pi^b(y)}, \\ \langle T b^a(x) b^b(y) \rangle &= -i \frac{\delta b^a(x)}{\delta J_b^b(y)}, \quad \langle T \pi^a(x) \pi^b(y) \rangle = -i \frac{\delta \pi^a(x)}{\delta J_\pi^b(y)}, \\ \langle T c^a(x) \bar{c}^b(y) \rangle &= i \frac{\delta c^a(x)}{\delta J_{\bar{c}}^b(y)}, \quad \langle T \xi^a(x) \bar{\xi}^b(y) \rangle = i \frac{\delta \xi^a(x)}{\delta J_{\bar{\xi}}^b(y)}. \end{aligned} \quad (24)$$

	$\Theta_{\lambda\nu}$	$\Omega_{\lambda\nu}$	$\Sigma_{\lambda\nu}$
$\Theta^{\mu\lambda}$	$\Theta^\mu_\nu$	0	$\Sigma^\mu_\nu$
$\Omega^{\mu\lambda}$	0	$\Omega^\mu_\nu$	0
$\Sigma^{\mu\lambda}$	$\Sigma^\mu_\nu$	0	$-\square\Theta^\mu_\nu$

TABLE I: Operator algebra fulfilled by  $\Theta$ ,  $\Omega$  and  $\Sigma$ .

It should be noticed that the functional derivatives satisfy the following property:

$$\frac{\delta^2}{\delta X_1^{g_1}(x)\delta X_2^{g_2}(y)} = (-1)^{g_1 \cdot g_2} \frac{\delta^2}{\delta X_2^{g_2}(y)\delta X_1^{g_1}(x)} , \quad (25)$$

where the upper indices,  $g_1$  and  $g_2$ , are the Faddeev-Popov charges ( $\Phi\Pi$ ) carried by the fields or currents,  $X_1^{g_1}$  and  $X_2^{g_2}$ , respectively. Due to the fact that the functional  $Z^c[J]$  (21) has ghost number zero, the “classical” sources  $J_i = (J_A^\mu, J_\phi^\mu, J_b, J_\pi, \bar{J}_c, \bar{J}_\xi, J_\xi)$  into the Legendre transformation (21), which relates the connected functional  $Z^c[J]$  and vertex functional  $\Gamma^{(0)}[\Phi]$ , have ghost numbers  $\Phi\Pi(J_i) = (0, 0, 0, 0, -1, 1, -1, 1)$ .

From the equations of motion we get,  $J_i \equiv J_i[\Phi_j]$ :

$$\begin{aligned} \frac{\delta\Gamma^{(0)}}{\delta A_\mu^a} &= \square\Theta^{\mu\nu}A_\nu^a + m\Sigma^{\mu\nu}\phi_\nu^a + \frac{1}{2}\partial^\mu b^a = -J_A^{a\mu} , \quad \frac{\delta\Gamma^{(0)}}{\delta b^a} = -\frac{1}{2}\partial^\mu A_\mu^a - \frac{\alpha}{2}b^a = -J_b^a , \\ \frac{\delta\Gamma^{(0)}}{\delta \phi_\mu^a} &= \square\Theta^{\mu\nu}\phi_\nu^a + m\Sigma^{\mu\nu}A_\nu^a + \frac{1}{2}\partial^\mu \pi^a = -J_\phi^{a\mu} , \quad \frac{\delta\Gamma^{(0)}}{\delta \pi^a} = -\frac{1}{2}\partial^\mu \phi_\mu^a - \frac{\beta}{2}\pi^a = -J_\pi^a , \\ \frac{\delta\Gamma^{(0)}}{\delta \bar{c}^a} &= \frac{1}{2}\square c^a = J_c^a , \quad \frac{\delta\Gamma^{(0)}}{\delta \bar{\xi}^a} = \frac{1}{2}\square \xi^a = J_\xi^a , \end{aligned} \quad (26)$$

where by solving these equations of motion (26) so as to express,  $\Phi_i \equiv \Phi_i[J_j]$ , and adopting the algebra fulfilled by the operators  $\Theta_{\mu\nu}$ ,  $\Omega_{\mu\nu}$  and  $\Sigma_{\mu\nu}$  displayed in Table I, it is found that:

$$\begin{aligned} A_\mu^a &= -\left\{ \frac{1}{\square + m^2}\Theta_{\mu\nu} - \frac{2\alpha}{\square}\Omega_{\mu\nu} \right\} J_A^{a\nu} + \frac{m}{\square(\square + m^2)}\Sigma_{\mu\nu}J_\phi^{a\nu} + \frac{2}{\square}\partial_\mu J_b^a , \\ \phi_\mu^a &= -\left\{ \frac{1}{\square + m^2}\Theta_{\mu\nu} - \frac{2\beta}{\square}\Omega_{\mu\nu} \right\} J_\phi^{a\nu} + \frac{m}{\square(\square + m^2)}\Sigma_{\mu\nu}J_A^{a\nu} + \frac{2}{\square}\partial_\mu J_\pi^a , \\ b^a &= -\frac{2}{\square}\partial_\mu J_A^{a\mu} , \quad \pi^a = -\frac{2}{\square}\partial_\mu J_\phi^{a\mu} , \quad c^a = \frac{2}{\square}J_c^a , \quad \xi^a = \frac{2}{\square}J_\xi^a . \end{aligned} \quad (27)$$

Now, by substituting the fields solutions, presented above in Eq.(27), into those ones in Eq.(24), the tree-level propagators are given by:

$$\begin{aligned} \langle TA_\mu^a(x)A_\nu^b(y) \rangle &= i\delta^{ab} \left\{ \frac{1}{\square + m^2}\Theta_{\mu\nu} - \frac{2\alpha}{\square}\Omega_{\mu\nu} \right\} \delta^3(x-y) , \\ \langle T\phi_\mu^a(x)\phi_\nu^b(y) \rangle &= i\delta^{ab} \left\{ \frac{1}{\square + m^2}\Theta_{\mu\nu} - \frac{2\beta}{\square}\Omega_{\mu\nu} \right\} \delta^3(x-y) , \\ \langle TA_\mu^a(x)\phi_\nu^b(y) \rangle &= -i\delta^{ab} \frac{m}{\square(\square + m^2)}\Sigma_{\mu\nu}\delta^3(x-y) , \\ \langle TA_\mu^a(x)b^b(y) \rangle &= -i\delta^{ab} \frac{2}{\square}\partial_\mu \delta^3(x-y) , \quad \langle T\phi_\mu^a(x)\pi^b(y) \rangle = -i\delta^{ab} \frac{2}{\square}\partial_\mu \delta^3(x-y) , \\ \langle Tb^a(x)b^b(y) \rangle &= 0 , \quad \langle T\pi^a(x)\pi^b(y) \rangle = 0 , \\ \langle Tc^a(x)\bar{c}^b(y) \rangle &= i\delta^{ab} \frac{2}{\square}\delta^3(x-y) , \quad \langle T\xi^a(x)\bar{\xi}^b(y) \rangle = i\delta^{ab} \frac{2}{\square}\delta^3(x-y) , \end{aligned} \quad (28)$$

where, assuming

$$\delta^3(x-y) = \int \frac{d^3k}{(2\pi)^3} e^{-ik(x-y)} , \quad (29)$$

the propagators in momenta space read:

$$\langle A_\mu^a(k) A_\nu^b(k) \rangle = -i\delta^{ab} \left\{ \frac{1}{k^2 - m^2} \left( \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) - \frac{2\alpha}{k^2} \left( \frac{k_\mu k_\nu}{k^2} \right) \right\}, \quad (30)$$

$$\langle \phi_\mu^a(k) \phi_\nu^b(k) \rangle = -i\delta^{ab} \left\{ \frac{1}{k^2 - m^2} \left( \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) - \frac{2\beta}{k^2} \left( \frac{k_\mu k_\nu}{k^2} \right) \right\}, \quad (31)$$

$$\langle A_\mu^a(k) \phi_\nu^b(k) \rangle = -\delta^{ab} \frac{m}{k^2(k^2 - m^2)} \epsilon_{\mu\rho\nu} k^\rho, \quad (32)$$

$$\langle A_\mu^a(k) b^b(k) \rangle = \delta^{ab} \frac{2}{k^2} k_\mu, \quad \langle \phi_\mu^a(k) \pi^b(k) \rangle = \delta^{ab} \frac{2}{k^2} k_\mu, \quad (33)$$

$$\langle b^a(k) b^b(k) \rangle = 0, \quad \langle \pi^a(k) \pi^b(k) \rangle = 0, \quad (34)$$

$$\langle c^a(k) \bar{c}^b(k) \rangle = -i\delta^{ab} \frac{2}{k^2}, \quad \langle \xi^a(k) \bar{\xi}^b(k) \rangle = -i\delta^{ab} \frac{2}{k^2}. \quad (35)$$

## B. Unitarity and causality

We will now discuss the spectrum and tree-level unitarity of the model. By coupling the propagators to external currents,  $\mathcal{J}_{\Phi_i}^a = (\mathcal{J}_A^{a\mu}, \mathcal{J}_\phi^{a\mu}, \mathcal{J}_b^a, \mathcal{J}_\pi^a, \mathcal{J}_c^a, \mathcal{J}_{\bar{c}}^a, \mathcal{J}_\xi^a, \mathcal{J}_{\bar{\xi}}^a)$ , compatible with the symmetries of the model, and then taking the imaginary part of the residues of the current-current amplitudes,  $\mathcal{A}_{\Phi_i\Phi_j}$ , at the poles, we can probe the necessary conditions for unitarity (positive imaginary part of the residues of the transition amplitudes,  $\Im \text{Res } \mathcal{A}_{\Phi_i\Phi_j} > 0$ , as a consequence of the  $S$ -matrix be unitary) at the tree-level and count the degrees of freedom described by the fields,  $\Phi_i^a = (A_\mu^a, \phi_\mu^a, b^a, \pi^a, c^a, \bar{c}^a, \xi^a, \bar{\xi}^a)$ . The current-current transition amplitudes in momentum space are written as:

$$\mathcal{A}_{\Phi_i\Phi_j} = \mathcal{J}_{\Phi_i}^{*a}(k) \langle \Phi_i^a(k) \Phi_j^b(k) \rangle \mathcal{J}_{\Phi_j}^b(k). \quad (36)$$

At this moment we will first analyze the case of the propagators of the vector fields  $A_\mu^a$  and  $\phi_\mu^a$ , given by Eqs.(30)–(32). The vector currents,  $\mathcal{J}_A^{a\mu}$  and  $\mathcal{J}_\phi^{a\mu}$ , can be expanded in terms of a three-dimensional complete basis in the momentum space as follows:

$$\mathcal{J}_A^{a\mu} = X_A^a k^\mu + Y_A^a \tilde{k}^\mu + Z_A^a \varepsilon^\mu \quad \text{and} \quad \mathcal{J}_\phi^{a\mu} = X_\phi^a k^\mu + Y_\phi^a \tilde{k}^\mu + Z_\phi^a \varepsilon^\mu, \quad (37)$$

fulfilling the current conservation conditions:

$$k_\mu \mathcal{J}_A^{a\mu} = 0 \quad \text{and} \quad k_\mu \mathcal{J}_\phi^{a\mu} = 0, \quad (38)$$

where  $k^\mu = (k^0, \vec{k})$ ,  $\tilde{k}^\mu = (k^0, -\vec{k})$  and  $\varepsilon^\mu = (0, \vec{\varepsilon})$  are linearly independent vectors satisfying the constraints:

$$k^\mu \varepsilon_\mu = \tilde{k}^\mu \varepsilon_\mu = 0 \quad \text{and} \quad \varepsilon^\mu \varepsilon_\mu = -1, \quad (39)$$

such that for a massive pole,  $k^\mu k_\mu = \tilde{k}^\mu \tilde{k}_\mu = m^2$ , and for a massless one,  $k^\mu k_\mu = \tilde{k}^\mu \tilde{k}_\mu = 0$ .

In the massive case ( $k^2 = m^2$ ), the momentum can be chosen as  $k^\mu = (m, \vec{0})$ , and by the current conservation conditions (38), the currents  $\mathcal{J}_A^{a\mu}$  and  $\mathcal{J}_\phi^{a\mu}$  are given by:

$$\mathcal{J}_A^{a\mu}|_{k^2=m^2} = Z_A^a(0, \vec{\varepsilon}) \quad \text{and} \quad \mathcal{J}_\phi^{a\mu}|_{k^2=m^2} = Z_\phi^a(0, \vec{\varepsilon}). \quad (40)$$

On the other hand, in the massless case ( $k^2 = 0$ ), the momentum chosen as  $k^\mu = (m, 0, m)$  together with the current conservation conditions (38) fix the currents  $\mathcal{J}_A^{a\mu}$  and  $\mathcal{J}_\phi^{a\mu}$  as below:

$$\mathcal{J}_A^{a\mu}|_{k^2=0} = (mX_A^a, Z_A^a, mX_A^a) \quad \text{and} \quad \mathcal{J}_\phi^{a\mu}|_{k^2=0} = (mX_\phi^a, Z_\phi^a, mX_\phi^a). \quad (41)$$

The current-current amplitudes for the vector fields  $A_\mu^a$  and  $\phi_\mu^a$  are given by:

$$\mathcal{A}_{AA} = \mathcal{J}_A^{*a\mu}(k) \langle A_\mu^a(k) A_\nu^b(k) \rangle \mathcal{J}_A^{b\nu}(k) = -i \frac{1}{k^2 - m^2} \mathcal{J}_A^{*a\mu} \mathcal{J}_A^a, \quad (42)$$

$$\mathcal{A}_{\phi\phi} = \mathcal{J}_\phi^{*a\mu}(k) \langle \phi_\mu^a(k) \phi_\nu^b(k) \rangle \mathcal{J}_\phi^{b\nu}(k) = -i \frac{1}{k^2 - m^2} \mathcal{J}_\phi^{*a\mu} \mathcal{J}_\phi^a, \quad (43)$$

$$\mathcal{A}_{A\phi} = \mathcal{J}_A^{*a\mu}(k) \langle A_\mu^a(k) \phi_\nu^b(k) \rangle \mathcal{J}_\phi^{b\nu}(k) = -\frac{m}{k^2(k^2 - m^2)} \epsilon_{\mu\rho\nu} \mathcal{J}_A^{*a\mu} k^\rho \mathcal{J}_\phi^{a\nu}, \quad (44)$$

where use has been made of the current conservation conditions (38). Analyzing the amplitudes above, it can be verified that the amplitudes  $\mathcal{A}_{AA}$  (42) and  $\mathcal{A}_{\phi\phi}$  (43) have single massive poles at  $k^2 = m^2$ , whereas the amplitude  $\mathcal{A}_{A\phi}$  (44) has two poles, a massive and a massless, at  $k^2 = m^2$  and  $k^2 = 0$ , respectively. Bearing in mind the currents  $\mathcal{J}_A^{a\mu}$  and  $\mathcal{J}_\phi^{a\mu}$  calculated at the poles  $k^2 = m^2$  (40) and  $k^2 = 0$  (41), the residues of the current-current amplitudes  $\mathcal{A}_{AA}$ ,  $\mathcal{A}_{\phi\phi}$  and  $\mathcal{A}_{A\phi}$  evaluated at their respective poles give rise to:

$$\text{Res } \mathcal{A}_{AA}|_{k^2=m^2} = i|Z_A^a|^2, \quad (45)$$

$$\text{Res } \mathcal{A}_{\phi\phi}|_{k^2=m^2} = i|Z_\phi^a|^2, \quad (46)$$

$$\text{Res } \mathcal{A}_{A\phi}|_{k^2=m^2} = 0 \quad \text{and} \quad \text{Res } \mathcal{A}_{A\phi}|_{k^2=0} = 0. \quad (47)$$

Therefore, by considering the imaginary part of the residues ( $\Im\text{Res}$ ) above, we get:

$$\Im\text{Res } \mathcal{A}_{AA}|_{k^2=m^2} = |Z_A^a|^2 > 0, \quad (48)$$

$$\Im\text{Res } \mathcal{A}_{\phi\phi}|_{k^2=m^2} = |Z_\phi^a|^2 > 0, \quad (49)$$

$$\Im\text{Res } \mathcal{A}_{A\phi}|_{k^2=m^2} = 0 \quad \text{and} \quad \Im\text{Res } \mathcal{A}_{A\phi}|_{k^2=0} = 0, \quad (50)$$

then, it can be concluded from (48) and (49) that the both vector fields,  $A_\mu^a$  and  $\phi_\mu^a$ , carry  $2(N^2 - 1)$  massive degrees of freedom with mass  $m$ , however, from (50) it follows that there are no massless degrees of freedom propagating associated to the vector fields.

Let us now analyze the propagators related to the fields  $b^a$ ,  $\pi^a$ ,  $c^a$ ,  $\bar{c}^a$ ,  $\xi^a$  and  $\bar{\xi}^a$ , given by Eqs.(33)–(35). The current-current amplitudes read:

$$\mathcal{A}_{Ab} = \mathcal{J}_A^{*a\mu}(k) \langle A_\mu^a(k) b^b(k) \rangle \mathcal{J}_b^b(k) = \frac{2}{k^2} k_\mu \mathcal{J}_A^{*a\mu} \mathcal{J}_b^a = 0, \quad (51)$$

$$\mathcal{A}_{\phi\pi} = \mathcal{J}_\phi^{*a\mu}(k) \langle \phi_\mu^a(k) \pi^b(k) \rangle \mathcal{J}_\pi^b(k) = \frac{2}{k^2} k_\mu \mathcal{J}_\phi^{*a\mu} \mathcal{J}_\pi^a = 0, \quad (52)$$

$$\mathcal{A}_{bb} = \mathcal{J}_b^{*a}(k) \langle b^a(k) b^b(k) \rangle \mathcal{J}_b^b(k) = 0, \quad \mathcal{A}_{\pi\pi} = \mathcal{J}_\pi^{*a}(k) \langle \pi^a(k) \pi^b(k) \rangle \mathcal{J}_\pi^b(k) = 0, \quad (53)$$

$$\mathcal{A}_{c\bar{c}} = \mathcal{J}_c^{*a}(k) \langle c^a(k) \bar{c}^b(k) \rangle \mathcal{J}_{\bar{c}}^b(k) = -i \frac{2}{k^2} \mathcal{J}_c^{*a} \mathcal{J}_{\bar{c}}^a, \quad \mathcal{A}_{\xi\bar{\xi}} = \mathcal{J}_\xi^{*a}(k) \langle \xi^a(k) \bar{\xi}^b(k) \rangle \mathcal{J}_{\bar{\xi}}^b(k) = -i \frac{2}{k^2} \mathcal{J}_\xi^{*a} \mathcal{J}_{\bar{\xi}}^a, \quad (54)$$

where the current conservation conditions (38) were applied in (51) and (52). Through the amplitudes displayed above, by considering their imaginary parts of the residues at the massless pole  $k^2 = 0$ :

$$\Im\text{Res } \mathcal{A}_{Ab}|_{k^2=0} = 0, \quad \Im\text{Res } \mathcal{A}_{\phi\pi}|_{k^2=0} = 0, \quad \Im\text{Res } \mathcal{A}_{bb}|_{k^2=0} = 0, \quad \Im\text{Res } \mathcal{A}_{\pi\pi}|_{k^2=0} = 0, \quad (55)$$

$$\Im\text{Res } \mathcal{A}_{c\bar{c}}|_{k^2=0} = -2 \mathcal{J}_c^{*a} \mathcal{J}_{\bar{c}}^a < 0 \quad \text{and} \quad \Im\text{Res } \mathcal{A}_{\xi\bar{\xi}}|_{k^2=0} = -2 \mathcal{J}_\xi^{*a} \mathcal{J}_{\bar{\xi}}^a < 0, \quad (56)$$

it shows that there are no massless modes propagating in the Lautrup-Nakanishi fields sector (55), nevertheless, from (56) we see that the massless propagating (negative norm state) Faddeev-Popov ghosts (antighosts)  $c^a$  and  $\xi^a$  ( $\bar{c}^a$  and  $\bar{\xi}^a$ ) carry, each of them,  $N^2 - 1$  degrees of freedom – taking care of the  $N^2 - 1$  spurious degrees of freedom stemming from the longitudinal sector of each vector field,  $A_\mu^a$  (30) and  $\phi_\mu^a$  (31).

From the results presented above, it can be concluded that the Jackiw-Pi model is free from tachyons and ghosts at the classical level. Nevertheless, to have full control of the unitarity at tree-level, it is still necessary to study the behaviour of the scattering cross sections in the limit of high center of mass energies, by analyzing the Froissart-Martin bound [16, 17].

### C. Ultraviolet and infrared dimensions

In order to establish the ultraviolet (UV) and infrared (IR) dimensions of any field,  $X$  and  $Y$ , we make use of the UV and IR asymptotical behaviour of their propagator,  $\Delta_{XY}(k)$ ,  $d_{XY}$  and  $r_{XY}$ , respectively:

$$d_{XY} = \overline{\deg}_k \Delta_{XY}(k) \quad \text{and} \quad r_{XY} = \underline{\deg}_k \Delta_{XY}(k), \quad (57)$$

where the upper degree  $\overline{\deg}_k$  gives the asymptotic power for  $k \rightarrow \infty$  whereas the lower degree  $\underline{\deg}_k$  gives the asymptotic power for  $k \rightarrow 0$ . The UV ( $d$ ) and IR ( $r$ ) dimensions of the fields,  $X$  and  $Y$ , are chosen to fulfill the following inequalities:

$$d_X + d_Y \geq 3 + d_{XY} \quad \text{and} \quad r_X + r_Y \leq 3 + r_{XY}. \quad (58)$$

Since the Landau gauge shall be adopted later, the UV and IR dimensions of all the fields are fixed assuming  $\alpha = \beta = 0$ . In order to fix the UV and IR dimensions of the vector fields  $A_\mu$  and  $\phi_\mu$ , use has been made of the propagators, (30), (31) and (32), together with the conditions (58), and the following conditions stem from:

$$2d_A \geq 1, \quad 2d_\phi \geq 1 \quad \text{and} \quad d_A + d_\phi \geq 0 \quad \longrightarrow \quad d_A = d_\phi = \frac{1}{2}; \quad (59)$$

$$2r_A \leq 3, \quad 2r_\phi \leq 3 \quad \text{and} \quad r_A + r_\phi \leq 2 \quad \longrightarrow \quad r_A = r_\phi = \frac{1}{2}. \quad (60)$$

From the propagators (33) and the conditions, (58), (59) and (60), we can fix the UV and IR dimensions of the Lautrup-Nakanishi fields,  $b$  and  $\pi$ , as follows:

$$d_A + d_b \geq 2 \quad \text{and} \quad d_A = \frac{1}{2} \quad \longrightarrow \quad d_b = \frac{3}{2}; \quad d_\phi + d_\pi \geq 2 \quad \text{and} \quad d_\phi = \frac{1}{2} \quad \longrightarrow \quad d_\pi = \frac{3}{2}; \quad (61)$$

$$r_A + r_b \leq 2 \quad \text{and} \quad r_A = \frac{1}{2} \quad \longrightarrow \quad r_b = \frac{3}{2}; \quad r_\phi + r_\pi \leq 2 \quad \text{and} \quad r_\phi = \frac{1}{2} \quad \longrightarrow \quad r_\pi = \frac{3}{2}. \quad (62)$$

The dimensions (UV and IR) of the Faddeev-Popov ghosts ( $c$  and  $\xi$ ) and antighosts ( $\bar{c}$  and  $\bar{\xi}$ ) are fixed, by considering the propagators (35), such that:

$$d_c + d_{\bar{c}} \geq 1 \quad \text{and} \quad d_\xi + d_{\bar{\xi}} \geq 1; \quad (63)$$

$$r_c + r_{\bar{c}} \leq 1 \quad \text{and} \quad r_\xi + r_{\bar{\xi}} \leq 1. \quad (64)$$

Also, assuming that the BRS operator  $s$  (6) is dimensionless and bearing in mind that the coupling constant  $g$  has dimension  $(\text{mass})^{\frac{1}{2}}$ , we get the following results for the ghosts, antighosts and  $\rho$  field:

$$d_c = -\frac{1}{2}, \quad d_{\bar{c}} = \frac{3}{2}, \quad d_\xi = -\frac{1}{2}, \quad d_{\bar{\xi}} = \frac{3}{2} \quad \text{and} \quad d_\rho = -\frac{1}{2}; \quad (65)$$

$$r_c = -\frac{1}{2}, \quad r_{\bar{c}} = \frac{3}{2}, \quad r_\xi = -\frac{1}{2}, \quad r_{\bar{\xi}} = \frac{3}{2} \quad \text{and} \quad r_\rho = -\frac{1}{2}. \quad (66)$$

Finally, through the action of the antifields (12), and the UV and IR dimensions of the fields fixed previously, it follows that

$$d_{A^*} = \frac{5}{2}, \quad d_{\phi^*} = \frac{5}{2}, \quad d_{\rho^*} = \frac{7}{2}, \quad d_{c^*} = \frac{7}{2} \quad \text{and} \quad d_{\xi^*} = \frac{7}{2}; \quad (67)$$

$$r_{A^*} = \frac{5}{2}, \quad r_{\phi^*} = \frac{5}{2}, \quad r_{\rho^*} = \frac{7}{2}, \quad r_{c^*} = \frac{7}{2} \quad \text{and} \quad r_{\xi^*} = \frac{7}{2}. \quad (68)$$

In summary, the UV and IR dimensions,  $d$  and  $r$  respectively, the ghost numbers,  $\Phi\Pi$ , of all fields are collected in Table II.

#### IV. SLAVNOV-TAYLOR IDENTITY, GHOST AND ANTIGHOST EQUATIONS AND WARD IDENTITIES

This subsection is devoted to establish the Slavnov-Taylor identity, ghost and antighost equations, and two hidden rigid symmetries. The BRS invariance of the action  $\Gamma^{(0)}$  (14) is expressed through the Slavnov-Taylor identity

$$\begin{aligned} \mathcal{S}(\Gamma^{(0)}) = & \text{Tr} \int d^3x \left\{ \frac{\delta\Gamma^{(0)}}{\delta A_\mu^*} \frac{\delta\Gamma^{(0)}}{\delta A^\mu} + \frac{\delta\Gamma^{(0)}}{\delta \phi_\mu^*} \frac{\delta\Gamma^{(0)}}{\delta \phi^\mu} + \frac{\delta\Gamma^{(0)}}{\delta \rho^*} \frac{\delta\Gamma^{(0)}}{\delta \rho} + \frac{\delta\Gamma^{(0)}}{\delta c^*} \frac{\delta\Gamma^{(0)}}{\delta c} + \frac{\delta\Gamma^{(0)}}{\delta \xi^*} \frac{\delta\Gamma^{(0)}}{\delta \xi} \right. \\ & \left. + b \frac{\delta\Gamma^{(0)}}{\delta \bar{c}} + \pi \frac{\delta\Gamma^{(0)}}{\delta \bar{\xi}} \right\} = 0, \end{aligned} \quad (69)$$

which translates, in a functional way, the invariance of the classical model under the BRS symmetry. It is suitable to define, for later use, the linearized Slavnov-Taylor ( $\mathcal{S}_{\Gamma^{(0)}}$ ) operator as below

$$\begin{aligned} \mathcal{S}_{\Gamma^{(0)}} = & \text{Tr} \int d^3x \left\{ \frac{\delta\Gamma^{(0)}}{\delta A_\mu^*} \frac{\delta}{\delta A^\mu} + \frac{\delta\Gamma^{(0)}}{\delta A^\mu} \frac{\delta}{\delta A_\mu^*} + \frac{\delta\Gamma^{(0)}}{\delta \phi_\mu^*} \frac{\delta}{\delta \phi^\mu} + \frac{\delta\Gamma^{(0)}}{\delta \phi^\mu} \frac{\delta}{\delta \phi_\mu^*} + \frac{\delta\Gamma^{(0)}}{\delta \rho^*} \frac{\delta}{\delta \rho} + \frac{\delta\Gamma^{(0)}}{\delta \rho} \frac{\delta}{\delta \rho^*} \right. \\ & \left. + \frac{\delta\Gamma^{(0)}}{\delta c^*} \frac{\delta}{\delta c} + \frac{\delta\Gamma^{(0)}}{\delta c} \frac{\delta}{\delta c^*} + \frac{\delta\Gamma^{(0)}}{\delta \xi^*} \frac{\delta}{\delta \xi} + \frac{\delta\Gamma^{(0)}}{\delta \xi} \frac{\delta}{\delta \xi^*} + b \frac{\delta}{\delta \bar{c}} + \pi \frac{\delta}{\delta \bar{\xi}} \right\}. \end{aligned} \quad (70)$$



Another identities, the ghost equations,

$$\mathcal{G}_I \Gamma^{(0)} \equiv \frac{\delta \Gamma^{(0)}}{\delta \bar{c}} + \partial^\mu \frac{\delta \Gamma^{(0)}}{\delta A^{*\mu}} = 0 , \quad (71)$$

$$\mathcal{G}_{II} \Gamma^{(0)} \equiv \frac{\delta \Gamma^{(0)}}{\delta \bar{\xi}} + \partial^\mu \frac{\delta \Gamma^{(0)}}{\delta \phi^{*\mu}} = 0 , \quad (72)$$

follow from the gauge-fixing conditions,

$$\frac{\delta \Gamma^{(0)}}{\delta b} = \partial^\mu A_\mu + \alpha b , \quad (73)$$

$$\frac{\delta \Gamma^{(0)}}{\delta \pi} = \partial^\mu \phi_\mu + \beta \pi , \quad (74)$$

and the Slavnov-Taylor identity (69), meaning that  $\Gamma^{(0)}$  depends on the antighosts,  $\bar{c}$  and  $\bar{\xi}$ , and the antifields,  $A^{*\mu}$  and  $\phi^{*\mu}$ , through the combinations

$$\tilde{A}_\mu^* = A_\mu^* + \partial_\mu \bar{c} \quad \text{and} \quad \tilde{\phi}_\mu^* = \phi_\mu^* + \partial_\mu \bar{\xi} . \quad (75)$$

The Jackiw-Pi model presents two antighost equations, they are listed as below:

$$\bar{\mathcal{G}}_I \Gamma^{(0)} \equiv \int d^3x \left\{ \frac{\delta \Gamma^{(0)}}{\delta c} - g \left[ \bar{c}, \frac{\delta \Gamma^{(0)}}{\delta b} \right] - g \left[ \bar{\xi}, \frac{\delta \Gamma^{(0)}}{\delta \pi} \right] \right\} = \bar{\Delta}_I , \quad (76)$$

$$\text{where } \bar{\Delta}_I \equiv -g \int d^3x \left\{ [A_\mu^*, A^\mu] + [\phi_\mu^*, \phi^\mu] + [\rho^*, \rho] - [c^*, c] - [\xi^*, \xi] + \alpha [\bar{c}, b] + \beta [\bar{\xi}, \pi] \right\} ; \quad (77)$$

$$\bar{\mathcal{G}}_{II} \Gamma^{(0)} \equiv \int d^3x \left\{ \frac{\delta \Gamma^{(0)}}{\delta \xi} - g \left[ \bar{\xi}, \frac{\delta \Gamma^{(0)}}{\delta b} \right] \right\} = \bar{\Delta}_{II} , \quad (78)$$

$$\text{where } \bar{\Delta}_{II} \equiv -g \int d^3x \left\{ [\phi_\mu^*, A^\mu] - [\xi^*, c] - \frac{\rho^*}{g} + \alpha [\bar{\xi}, b] \right\} . \quad (79)$$

It should be noticed, for the sake of further quantization [8], that the breakings,  $\bar{\Delta}_I$  and  $\bar{\Delta}_{II}$ , being nonlinear in the quantum fields will be subjected to renormalization. An interesting issue in Yang-Mills theories is that the Landau gauge [18] has very special features as compared to a generic linear gauge. This is due to the existence, besides the Slavnov-Taylor identity, of another identity, the antighost equation [19], which controls the dependence of the theory on the ghost  $c$ . In particular, this equation implies that the ghost field  $c$  and the composite  $c$ -field cocycles in the descent equations have vanishing anomalous dimension, allowing the algebraic proof [9] of the Adler-Bardeen nonrenormalization theorem [20] for the gauge anomaly. Back to the Jackiw-Pi model we are considering here, in the case of the general linear covariant gauges, (9) and (10), the right-hand sides of the equations, (76) and (78), are nonlinear in the quantum fields due to the presence of the terms,  $\int d^3x \alpha [\bar{c}, b]$  and  $\int d^3x \beta [\bar{\xi}, \pi]$ , and  $\int d^3x \alpha [\bar{\xi}, b]$ , respectively. Therefore, the breakings,  $\bar{\Delta}_I$  (77) and  $\bar{\Delta}_{II}$  (79) have to be renormalized, which could spoil the usefulness of the antighost equations, by this reason, bearing in mind later renormalization of the model, we adopt from now on the Landau gauge  $\alpha = \beta = 0$ .

As another feature of the Landau gauge, the following Ward identities for the rigid symmetries stem from the Slavnov-Taylor identity (69) and the antighost equations (76) and (78) with  $\alpha = \beta = 0$ :

$$\mathcal{W}_I^{\text{rig}} \Gamma^{(0)} = 0 , \quad \text{where}$$

$$\begin{aligned} \mathcal{W}_I^{\text{rig}} \equiv & -g \int d^3x \left\{ \left[ A^\mu, \frac{\delta}{\delta A^\mu} \right] + \left[ \phi^\mu, \frac{\delta}{\delta \phi^\mu} \right] + \left[ \rho, \frac{\delta}{\delta \rho} \right] + \left[ b, \frac{\delta}{\delta b} \right] + \left[ \pi, \frac{\delta}{\delta \pi} \right] + \left[ c, \frac{\delta}{\delta c} \right] + \left[ \xi, \frac{\delta}{\delta \xi} \right] + \right. \\ & \left. + \left[ \bar{c}, \frac{\delta}{\delta \bar{c}} \right] + \left[ \bar{\xi}, \frac{\delta}{\delta \bar{\xi}} \right] + \left[ A_\mu^*, \frac{\delta}{\delta A_\mu^*} \right] + \left[ \phi_\mu^*, \frac{\delta}{\delta \phi_\mu^*} \right] + \left[ \rho^*, \frac{\delta}{\delta \rho^*} \right] + \left[ c^*, \frac{\delta}{\delta c^*} \right] + \left[ \xi^*, \frac{\delta}{\delta \xi^*} \right] \right\} ; \end{aligned} \quad (80)$$

$$\mathcal{W}_{II}^{\text{rig}} \Gamma^{(0)} = 0 , \quad \text{where}$$

$$\mathcal{W}_{II}^{\text{rig}} \equiv -g \int d^3x \left\{ \left[ A^\mu, \frac{\delta}{\delta \phi^\mu} \right] + \left[ \pi, \frac{\delta}{\delta b} \right] + \left[ c, \frac{\delta}{\delta \xi} \right] + \left[ \bar{\xi}, \frac{\delta}{\delta \bar{c}} \right] + \left[ \phi_\mu^*, \frac{\delta}{\delta A_\mu^*} \right] + \left[ \xi^*, \frac{\delta}{\delta c^*} \right] + \frac{1}{g} \frac{\delta}{\delta \rho} \right\} . \quad (81)$$

	$A_\mu$	$\phi_\mu$	$\rho$	$b$	$\pi$	$c$	$\xi$	$\bar{c}$	$\bar{\xi}$	$A^{*\mu}$	$\phi^{*\mu}$	$\rho^*$	$c^*$	$\xi^*$	$g$	$m$
$d$	1/2	1/2	-1/2	3/2	3/2	-1/2	-1/2	3/2	3/2	5/2	5/2	7/2	7/2	7/2	1/2	1
$r$	1/2	1/2	-1/2	3/2	3/2	-1/2	-1/2	3/2	3/2	5/2	5/2	7/2	7/2	7/2	1/2	1
$\Phi\Pi$	0	0	0	0	0	1	1	-1	-1	-1	-1	-1	-2	-2	0	0

TABLE II: Ultraviolet dimension ( $d$ ), infrared dimension ( $r$ ) and ghost number ( $\Phi\Pi$ ).

### A. Operatorial algebra

All operators introduced previously satisfy the following off-shell algebra for any functional  $\mathcal{K}$  with even Faddeev-Popov charge:

1. Slavnov-Taylor operator identities:

$$\begin{aligned}
S_{\mathcal{K}}S(\mathcal{K}) &= 0 \quad \forall \mathcal{K}, \quad S_{\mathcal{K}}S_{\mathcal{K}} = 0 \text{ if } S(\mathcal{K}) = 0, \\
\frac{\delta S(\mathcal{K})}{\delta b} - S_{\mathcal{K}} \left( \frac{\delta \mathcal{K}}{\delta b} - \partial^\mu A_\mu \right) &= \mathcal{G}_I(\mathcal{K}), \quad \frac{\delta S(\mathcal{K})}{\delta \pi} - S_{\mathcal{K}} \left( \frac{\delta \mathcal{K}}{\delta \pi} - \partial^\mu \phi_\mu \right) = \mathcal{G}_{II}(\mathcal{K}), \\
\mathcal{G}_I S(\mathcal{K}) + S_{\mathcal{K}} \mathcal{G}_I(\mathcal{K}) &= 0, \quad \mathcal{G}_{II} S(\mathcal{K}) + S_{\mathcal{K}} \mathcal{G}_{II}(\mathcal{K}) = 0, \\
\bar{\mathcal{G}}_I S(\mathcal{K}) + S_{\mathcal{K}} (\bar{\mathcal{G}}_I(\mathcal{K}) - \bar{\Delta}_I) &= \mathcal{W}_I(\mathcal{K}), \quad \mathcal{W}_I S(\mathcal{K}) - S_{\mathcal{K}} \mathcal{W}_I(\mathcal{K}) = 0, \\
\bar{\mathcal{G}}_{II} S(\mathcal{K}) + S_{\mathcal{K}} (\bar{\mathcal{G}}_{II}(\mathcal{K}) - \bar{\Delta}_{II}) &= \mathcal{W}_{II}(\mathcal{K}), \quad \mathcal{W}_{II} S(\mathcal{K}) - S_{\mathcal{K}} \mathcal{W}_{II}(\mathcal{K}) = 0;
\end{aligned} \tag{82}$$

2. Other identities:

$$\begin{aligned}
\bar{\mathcal{G}}_I^a (\bar{\mathcal{G}}_I^b(\mathcal{K}) - \bar{\Delta}_I^b) + \bar{\mathcal{G}}_I^b (\bar{\mathcal{G}}_I^a(\mathcal{K}) - \bar{\Delta}_I^a) &= 0, \quad \mathcal{W}_I^a \mathcal{W}_I^b(\mathcal{K}) - \mathcal{W}_I^b \mathcal{W}_I^a(\mathcal{K}) = 0, \\
\bar{\mathcal{G}}_I^a \mathcal{W}_I^b(\mathcal{K}) - \mathcal{W}_I^a (\bar{\mathcal{G}}_I^b(\mathcal{K}) - \bar{\Delta}_I^b) &= 0, \\
\bar{\mathcal{G}}_I^a (\bar{\mathcal{G}}_{II}^b(\mathcal{K}) - \bar{\Delta}_{II}^b) + \bar{\mathcal{G}}_{II}^a (\bar{\mathcal{G}}_I^b(\mathcal{K}) - \bar{\Delta}_I^b) &= 0, \quad \bar{\mathcal{G}}_I^a \mathcal{W}_{II}^b - \mathcal{W}_{II}^a (\bar{\mathcal{G}}_I^b(\mathcal{K}) - \bar{\Delta}_I^b) = 0, \\
\bar{\mathcal{G}}_{II}^a \mathcal{W}_I^b(\mathcal{K}) - \mathcal{W}_I^a (\bar{\mathcal{G}}_{II}^b(\mathcal{K}) - \bar{\Delta}_{II}^b) &= 0, \quad \mathcal{W}_I^a \mathcal{W}_{II}^b(\mathcal{K}) - \mathcal{W}_{II}^a \mathcal{W}_I^b(\mathcal{K}) = 0, \\
\bar{\mathcal{G}}_{II}^a (\bar{\mathcal{G}}_{II}^b(\mathcal{K}) - \bar{\Delta}_{II}^b) + \bar{\mathcal{G}}_{II}^b (\bar{\mathcal{G}}_{II}^a(\mathcal{K}) - \bar{\Delta}_{II}^a) &= 0, \quad \mathcal{W}_{II}^a \mathcal{W}_{II}^b(\mathcal{K}) - \mathcal{W}_{II}^b \mathcal{W}_{II}^a(\mathcal{K}) = 0, \\
\bar{\mathcal{G}}_{II}^a \mathcal{W}_{II}^b(\mathcal{K}) - \mathcal{W}_{II}^a (\bar{\mathcal{G}}_{II}^b(\mathcal{K}) - \bar{\Delta}_{II}^b) &= 0, \\
[\mathcal{P}_\mu, \Theta] &= 0 \quad \forall \Theta \in \{S_{\mathcal{K}}, \mathcal{G}_I, \mathcal{G}_{II}, \bar{\mathcal{G}}_I, \bar{\mathcal{G}}_{II}, \mathcal{W}_I, \mathcal{W}_{II}, \mathcal{P}_\mu\},
\end{aligned} \tag{83}$$

where  $\mathcal{P}_\mu$  is the Ward operator associated to translations:

$$\mathcal{P}_\mu = \sum_{\varphi} \text{Tr} \int d^3x \partial_\mu \varphi \frac{\delta}{\delta \varphi}, \tag{84}$$

and  $\varphi$  are all the fields contained in the action (14). The first group of identities involving the Slavnov-Taylor operator given by (82) are those which yield the conditions (the well-known Wess-Zumino consistency condition is one of them) to be satisfied by the quantum breaking of the Slavnov-Taylor identity (69) allowed by the Quantum Action Principle [9].

## V. CONCLUSIONS

The Jackiw-Pi model which generates a mass gap preserving parity in three space-time dimensions was presented here. The BRS symmetry of the model was established and all the difficulties found out in the literature concerning the gauge-fixing were by-passed. At the tree-level the propagators were computed, the spectrum consistency (causality and unitarity) has been verified and we conclude that the Jackiw-Pi model are free from tachyons and ghosts. By the asymptotical behaviour of the propagators together with the BRS transformations, the ultraviolet and infrared dimensions of all the fields were fixed. Also, BRS invariance and Slavnov-Taylor identity together with the antighost equations, in the Landau gauge, allowed to find out two rigid symmetries, moreover, the operatorial algebra which

defines the model has been presented. An important issue to notice is that, as we have shown, the Jackiw-Pi even-parity mass term is BRS invariant up to a total derivative, *i.e.*, it is not local BRS invariant. Therefore, it could be conjectured that, at the quantum level, the  $\beta$ -function associated to the mass parameter  $m$ ,  $\beta_m$ , should be zero,  $\beta_m = 0$  [13, 14]. Moreover, this fact would indicate the perturbatively ultraviolet finiteness of the Jackiw-Pi model, which is now under investigation [8] in the framework of the algebraic renormalization scheme.

### Acknowledgements

In honor of the 70th birthday of Prof. Olivier Piguet. Also, the author dedicates this work to his kids, Vittoria and Enzo, and to his mother, Victoria. Thanks are also due to Daniel H.T. Franco for discussions and encouragement.

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